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EVALUATION OF HEAT TRANSFER PERFORMANCES OF ROUGH SURFACES FROM EXPERIMENTAL INVESTIGATION IN ANNULAR CHANNELS

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Abstract—A new method for the transformation of turbulent Stanton numbers measured in the annular channel geometry was developed. The method is based on the solution of the simplified differential energy transport equation of the turbulent flow. To test this method, different heat-transfer calculations in the channels with smooth and rough surfaces are carried out. For comparison some other transformation procedures are also applied. The thermal performances of a particular roughness evaluated from the single rod experimental investigation, are used in a computer code to calculate the temperature distribution of a 37 rod bundle. The analytical predictions are compared with the experimental results.

NOMENCLATURE

a, thermal diffusivity;
c_p, specific heat;
c, constant;
d, diameter;
d_h, hydraulic diameter;
f, friction factor;
G, roughness function for heat transport;
h, rib height;
h^+, dimensionless rib height or local rib Reynolds number $[e \cdot u_\tau/v]$;
k, heat conductivity;
p, pitch of the ribs;
P, pressure;
Pr, Prandtl number;
Pr_t, turbulent Prandtl number;
$\dot{q}''$, heat flux;
r, radius;
r^+, dimensionless radius $[r \cdot u_\tau/v]$;
R, roughness function of momentum transport;
Re, Reynolds number;
St, Stanton number;
Stx, Stanton number multiplier;
T, absolute temperature (time mean);
T', temperature fluctuation;
T^+, dimensionless temperature defined by equation (9);
u, axial (time mean) velocity;
u', axial velocity fluctuation;
u^+, dimensionless axial velocity $[u/u_\tau]$;
u_τ, shear stress velocity $[(\tau_w/\rho)^{1/2}]$;
v, radial velocity (time mean);
v', radial velocity fluctuation;
w, rib width;
x, axial distance;
y, distance from the channel wall;
y^+, dimensionless wall distance $(y \cdot u_\tau/v)$.

Greek symbols

ρ, density;
v, kinematic viscosity;
μ, dynamic viscosity;
τ, shear stress;
Δ, difference;
ε_H, eddy diffusivity of heat;
ε_M, eddy diffusivity of momentum.

Indices

1, inner channel wall or transformed value for inner channel;
2, outer channel wall or transformed value for outer channel;
b, bulk;
c, calculated,
in, inlet;
m, maximum velocity position;
o, zero shear position;
s, smooth;
t, tube;
w, wall;
$\bar{}$, mean value over the cross section.

1. INTRODUCTION

1.1. *Description of the problem*

THE ANNULAR test section is a very convenient form of simple experimental channel for the fluid flow and heat-transfer investigations. Most of the experiments for the determination of thermohydraulic performances of rough surfaces were carried out in this channel geometry. In order to apply the annular experimental results to the more complicated channel geometries, these results must be transformed, considering changes in:

General flow conditions;
Channel geometry;

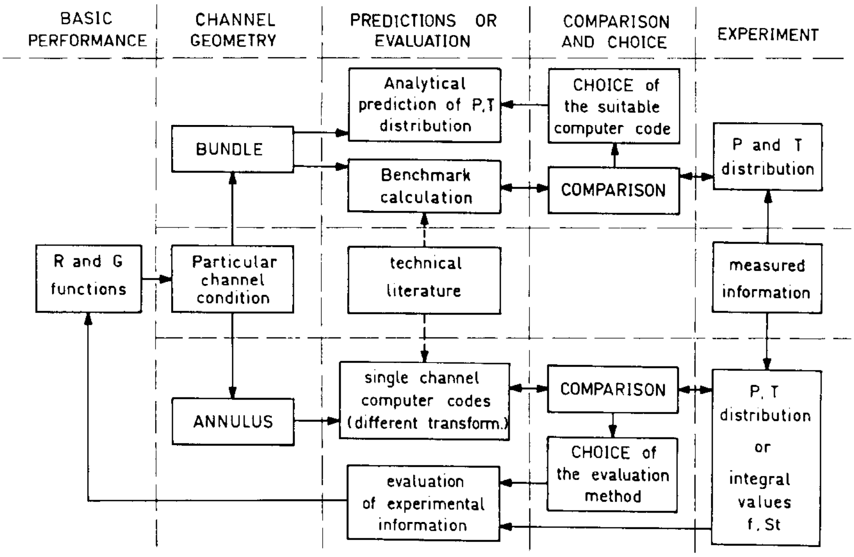


FIG. 1. A consistent EIR approach for evaluating the performances of rough surfaces and their consideration under different channel geometries.

Surface quality of the walls (proportion of rough to smooth surfaces in the channel);
Heat fluxes on the walls or proportion of heated to unheated surfaces.

The usual methods proposed for handling the experimental results do not make full allowance for the differences between experimental and actual channel. Therefore the direct transformation of the results is not generally possible. The methods attempt to reduce the experimental results to the uniform boundary conditions of the experimental channel. Unfortunately, for the asymmetric velocity profiles typical of annulus geometries, the accurate transformation procedure becomes more difficult. In case of inner rough and outer smooth wall surfaces (normal experimental situation) this asymmetry is more pronounced. Additionally, the similarity of the velocity and temperature profiles cannot be assumed due to the unilateral (inner wall) heating.

1.2. *Choice of a consistent approach for evaluating the performance of rough surfaces*

Contemporary to the experimental investigation of different rough surfaces in an annular channel geometry, suitable methods for the evaluation of the measured results were studied and compared. As a result of these investigations at EIR, the clear need for a consistent evaluation approach was acknowledged. Furthermore, to simplify the comparison of the experimental results of different investigators, the standardization of the evaluation methods is strongly recommended.

The general approach to evaluate the thermohydraulic performances of rough surfaces was worked out and published in 1974 [1]. This basic approach was recently reviewed and adapted at reasonable experimental costs considering that the main purpose of the transformation procedure is to achieve an acceptable accuracy in predicting the temperature and pressure distributions in rod bundles. This new approach is presented in Fig. 1. The evaluation of the thermohydraulic performances of the rough surfaces is based on the roughness functions R and G. The definition of these fundamental functions which provide local boundary conditions to the flow field were reviewed by Lewis who also summarised their limitations [2, 3]. Lewis and other authors assumed that these functions are invariant with the change in the core flow geometry. The results of the experimental investigations transformed with the usual methods don't confirm this. Assuming that the R and G functions are adjusted to the core flow geometry or in the case of channels where this channel geometry effect can be neglected, these functions provide the boundary conditions for the analytical solutions of the core flow in any system. That is, once these functions are known, the friction factors and Stanton numbers may be determined for any channel by solving the equations of the core momentum and heat transport matched to these boundary values. To obtain the R and G functions from the experimental investigation in annular channels a decision about the standard method of the evaluation of these experimental results should be made. The choice of this method should be based on the comparison of the R and G values (or transformed friction factors and Stanton numbers) obtained by different transformation methods and on their experimental confirmation. Earlier investigations show an acceptable agreement in the transformed friction factors, but, on the other hand, considerable differences between the results of the different evaluations of the heat transfer experimental data. This leads to the conclusion that further investigation and improvement of the transformation methods are needed before the final choice for the standard method can be made. The R and G functions can then be inserted in the computer codes for bundle geometry calculations. Different computer codes should be used for extensive bench mark calculations. Based on the comparison of the different predictions with the results measured in bundles these codes should be continuously improved. The benchmark calculations together with the experimental verifications are of a decisive importance for the final choice of an appropriate computer code.

1.3. *Review of the recent established methods for Stanton number transformation in annuli*

A basic method for the transformation of experimental results obtained in annular channels was proposed by Hall [4]. This method is based on the measured velocity and temperature profiles. The influence of the rough inner wall of the annulus on the heat transfer characteristics of the flow is incorporated directly into the Stanton number as an integral quantity.

Instead of solving the differential energy equation, the principle of superposition based on the known temperature distribution in the channel was used. To avoid the measurements of the temperature profile many approximate solutions have been proposed by different authors. A survey of these methods up to about 1970 was given by Wilkie [5], Kjellström [6] and Markòczy [7]. Most of these methods were experimentally checked and gave satisfactory results at least in a limited range of the experimental investigations. Few of them were used as practical semiempirical equations introducing some additional correction factors.

In the following, the methods for the Stanton number transformation recently applied by different investigators are shortly described.

Hall–Kjelleström transformation [6]. A combination of the simplified "equivalent conductivity" (assuming eddy diffusivity of heat is constant and predominant out of thermal boundary layer) and "laminar sublayer" concept is used to calculate the transformed Stanton number for a unilaterally heated annulus. To obtain the roughness function G, the heat transfer similarity law of Dipprey and Sabersky [8] is applied.

Rapier transformation [9]. In this transformation, the assumption of a linear shear stress distribution together with the Reynolds analogy is applied. The investigation over the channel is made assuming the eddy diffusivity of heat to be constant. The variation of the heat flux between inner and outer wall is taken into account. The effect of the curvature of the walls is approximated by an additional correction factor. The original method gives only the transformed Stanton number, but the G values can be calculated according to the heat transfer similarity law of Dipprey and Sabersky.

Nathan–Pirie transformation [10]. The Nathan–Pirie method is based on the empirical models obtained by fitting the experimental results for different transverse and helically ribbed surfaces in limited Reynolds number and friction factor ranges ($0.04 < f_1 < 0.16$; $5 \cdot 10^4 < Re < 3 \cdot 10^5$). The transformed Stanton numbers can be used to calculate the G function from the similarity law.

Dalle Donne–Meyer transformation [11]. It is assumed that, for the unilaterally heated annulus, the dimensionless temperature profile can be represented by a logarithmic law. This logarithmic temperature distribution is matched to give the measured Stanton number (alternatively the measured temperature of the outer wall can be fitted in the logarithmic distribution). The value of the G function is defined as the additive constant of the logarithmic law. In order to compare the transformed Stanton numbers with those of other methods, they are calculated considering the logarithmic temperature profile in the inner region of the annulus.

2. TRANSFORMATION PRINCIPLE

2.1. *Application of the annular experimental results to the bundle conditions*

Important differences can be observed when the rod bundle and the simple single channel experimental conditions are compared. It should be mentioned that the changes in channel geometry, heating and surface quality of the walls are considered here. The effect of the differences in Prandtl and Biot numbers, which are also very important, are not discussed in this report.

Two methods of application of the single rod experimental results to the rod bundle geometry are recently in use.

Direct method. The transformed experimental friction factors and Stanton numbers obtained in single channel geometries for a particular roughness shape can be directly applied in other more complex channel geometries, assuming that the equal hydraulic diameter concept (in fact the relative value h/d_{h_1}) is valid. The measurements with the same rough rod, but different diameter ratios of the test section show, particularly at low Reynolds numbers [12], significant deviations in the transformed quantities for equal h/d_{h_1} values. In practice, this method gives acceptable results if the "annular" concept can be applied to the bundle geometry and the ratios r_0/r_1 of the two geometries (experimental and "equivalent" annulus geometry of the bundle) are approximately equal. Generally the hydraulic diameter approach is only an approximative solution. The roughness performances are very strongly dependent on the h/d_{h_1} values but this parameter does not completely describe the changes occurring in the channel geometry. In spite of all, this method should be preferred if one wants to keep the computer codes for bundle calculations as simple as possible.

Use of roughness functions R *and* G. A more general transformation of the experimental results to the bundle conditions is achieved by introducing the roughness functions R and G as the boundary conditions for the rough surfaces. The comparisons

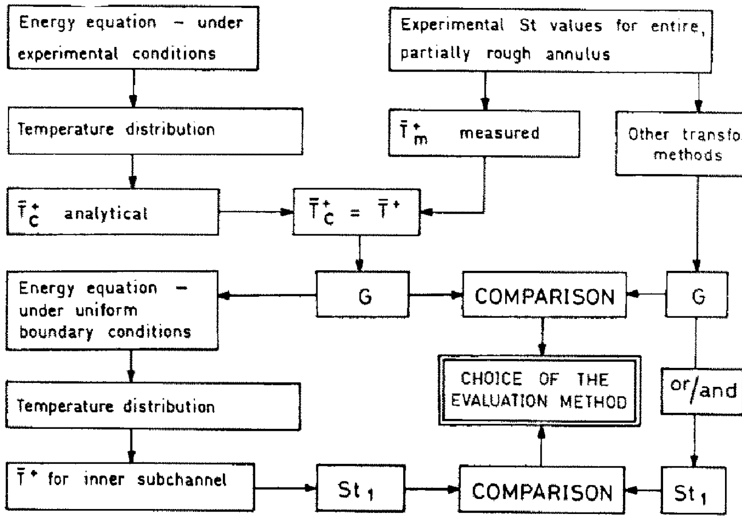


FIG. 2. Application of the new transformation method to the annular channel.

between the R and G values obtained in different channel geometries (tubes, flat walls, annular channels) by the recent evaluation methods, show a considerable scatter of the results. These differences can be explained by the great sensitivity of the R and G values to the accuracy of the measurements, but also, it can be assumed, by influences of the flow thickness y_0^+ and the wall curvature. Baumann and Rehme [13] carried out an extensive study of the experimental friction results for rectangular ribs. They introduced an additional explicit parameter h/y_0 for the correlation of the R function. Due to the connection between the roughness functions (similarity law) an important effect of this parameter on the G function can be assumed. The influence of the relative flow thickness y_0/h on the R and G functions was earlier observed in an experimental investigation of different other roughness shapes [14]. Unfortunately not enough experimental data are available to derive the corresponding correlations. Considering the assumptions made in the recent transformation procedures it cannot be excluded that the differences in the R and G values are due to the imperfections of these methods. More accurate solutions of the differential equations of the core flow can probably reduce these differences to a practically negligible amount.

2.2. *General requirements for the transformation methods*

To reach a satisfactory prediction of the pressure and temperature distributions in a rod bundle, the following requirements to a transformation method are regarded as very important:

Consistency and compatibility of the method. The same transformation procedure should be used for the evaluation of the experimental results and of the analytical predictions;
Consideration and full inclusion of the different boundary conditions typical of the different geometries;
Solution of the core flow differential equations for the different channel geometries.

None of the usual transformation methods can fulfil all those requirements.

2.3. *Principle of the new transformation method and its application to the annular channel*

The new transformation method is based on the solution of the differential equations of the core flow obtained by the "eddy diffusivity" concept. In these solutions the particular boundary conditions of the channel geometry are taken into account to obtain the corresponding dimensionless velocity and temperature distributions. Integrating these profiles over the flow cross section the mean dimensionless values of velocity and temperature are determined. Then, the corresponding friction factor and Stanton number can be calculated considering the well known connections between these integral performances and the mean values of dimensionless velocity and temperature.

Because of the considerable differences between the results of the transformed Stanton numbers, the investigations were started in order to improve the evaluation of experimental heat-transfer results obtained in an annular channel geometry. It is therefore assumed that the momentum transport equation is already solved and all important quantities (radius of zero shear surface, R function, velocity distribution and friction factor) are known. The value of the G function is determined matching the analytical solution of the core flow energy transport with the measured Stanton number. This value is then used to calculate the transformed Stanton number in an annulus with the equal inner subchannel and the following uniform boundary conditions:

The same roughness form on both walls of the annulus;

Both walls heated, where the ratio of the heat fluxes is determined from the condition of adiabatic zero shear plane,

$$\frac{\partial T}{\partial r} = 0, \quad \text{at } r = r_0.$$

The calculation flow diagram for the application of the method to the annular channel is presented in Fig. 2.

3. HEAT TRANSFER EQUATIONS FOR ANNULUS

3.1. *Basic energy equation*

To simplify the analytical treatment, the following basic conditions and assumption are considered:

Steady state, fully developed turbulent and incompressible flow;
Uniform heat flux;
Symmetric channel geometry (no eccentricity therefore two dimensional problem (x, r));
All important quantities (i.e. velocity, fluid properties, pressure and temperature) are separated in a time mean value and a fluctuating component;
All mean quantities are constant in the circumferential direction;
The mean velocity in other than axial direction is zero;
The fluid properties across the channel and within one axial calculation step are assumed to be constant;
The momentum equation is already solved and the radial distributions of velocity, eddy diffusivity of momentum and shear stress are known;
The boundary conditions at the inner, roughened wall are:

$$r = r_1, u = 0, \quad v = 0$$
$$u' = 0, \quad v' = 0$$
$$T = T_{w1}, \quad \dot{q}'' = \dot{q}''_{w1} = -k\frac{\partial T}{\partial r}$$
$$r = r_1 + h, u^+ = R, \quad T^+ = G;$$

(Similar set of conditions is used for the outer wall if the transformation to the uniform boundary conditions is accomplished.)
The boundary condition at the outer wall $(r = r_2)$ are:

Smooth wall surface

$$u = 0, \qquad v = 0,$$
$$u' = 0, \qquad v' = 0,$$
$$T = T_{w2}, \quad \dot{q}'' = 0, \quad \frac{\partial T}{\partial r} = 0,$$

(adiabatic outer surface, therefore heat losses neglected).

The energy equation without viscous dissipation can then be written:

$$c_p\rho u\frac{\partial T}{\partial x} - \frac{1}{r}\frac{\partial}{\partial r}\left[r\rho c_p\left(a\frac{\partial T}{\partial r} - \overline{v'T'}\right)\right] = 0. \quad (1)$$

3.2. *Eddy diffusivity concept*

The main problem to solve the energy equation (1) is to find an acceptable relation for the fluctuation term $\overline{v'T'}$, which governs the heat transport mechanism. One of the suitable approximations is to introduce the wall known eddy diffusivity concept. According to its original definition, this model is restricted to the flows with symmetric velocity profiles. For asymmetric profiles the displacement between the zero shear surface and the surface of the maximum velocity is caused by the diffusion of turbulence energy across the zero shear surface. Although this effect can only be described with more sophisticated models of turbulence the usefulness of the approximation with the eddy diffusivity model was confirmed by many authors for momentum [15–17] and heat transport [18]. With this concept a new fundamental analogy between heat and momentum transport is obtained allowing the energy transport across the zero surface to be considered.

If the eddy diffusivity of heat (eddy conductivity) defined by equation (2):

$$\varepsilon_H \equiv -\overline{v'T'}\bigg/\left(\frac{\partial T}{\partial r}\right), \quad (2)$$

is considered, the equation (1) can be rewritten:

$$c_p\rho u\frac{\partial T}{\partial x} - \frac{1}{r}\frac{\partial}{\partial r}\left[r\rho c_p(a+\varepsilon_H)\frac{\partial T}{\partial r}\right] = 0. \quad (3)$$

The eddy diffusivity of momentum (eddy viscosity) is assumed here to be known, therefore the introduction of its correlation with the eddy conductivity is very useful:

$$\frac{\varepsilon_M}{\varepsilon_H} \equiv Pr_t. \quad (4)$$

For the fluids with Prandtl numbers close to 1, an already simple expression for the turbulent Prandtl number Pr_t leads to satisfactory results.

3.3. *Heat flux distribution*

The direct solution of equation (3) can be bypassed using the following expression for the heat flux in the radial direction:

$$\dot{q}''(r) \equiv -\rho c_p(a+\varepsilon_H)\frac{\partial T}{\partial r}. \quad (5)$$

The equation (3) changes to equation (6):

$$\rho c_p u\frac{\partial T}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\dot{q}'') = 0, \quad (6)$$

and the integration of this equation, gives:

$$r\dot{q}'' = -\int \rho c_p ur\frac{\partial T}{\partial x}\,\mathrm{d}r + c. \quad (7)$$

The values ρ, c_p and $\partial T/\partial x$ do not change in the radial direction. Considering constant properties and the fully developed flow conditions,

$$\frac{\partial}{\partial x}\left(\frac{\partial T^+}{\partial r}\right) = 0 = \frac{\partial}{\partial r}\left(\frac{\partial T^+}{\partial x}\right), \quad (8)$$

where the dimensionless temperature is defined by

$$T^+ \equiv (T_w - T)\frac{\rho u_\tau c_p}{\dot{q}''_w}, \tag{9}$$

the equation (7) reduces to:

$$r\dot{q}''(r) = -\rho c_p \frac{\partial T}{\partial x}\int ur\,dr + c. \tag{10}$$

With the boundary condition of the inner wall of the annulus the heat flux distribution in the radial direction is obtained

$$\dot{q}''(r) = \dot{q}''_{1,w}\frac{r_1}{r} - \frac{\rho c_p}{r}\frac{\partial T}{\partial x}\int_{r_1}^{r} ur\,dr. \tag{11}$$

Integrating equation (11) over the whole channel, the axial temperature gradient in case of a heated outer wall is determined:

$$\frac{\partial T}{\partial x} = \frac{2(\dot{q}''_{1,w}r_1 - \dot{q}''_{2,w}r_2)}{\rho c_p \bar{u}(r_2^2 - r_1^2)}. \tag{12}$$

Combination of equations (11) and (12) leads to the following final equation of the heat flux distribution:

$$\frac{\dot{q}''(r)}{\dot{q}''_{1,w}} = \frac{r_1}{r}\left[1 - \frac{2\left(1 - \dfrac{r_2\dot{q}''_{2,w}}{r_1\dot{q}''_{1,w}}\right)}{\bar{u}(r_2^2 - r_1^2)}\cdot\int_{r_1}^{r} ur\,dr\right]. \tag{13}$$

3.4. *Radial temperature distribution*

If equations (4) and (5) are combined and the dimensionless forms of radius and temperature, related to the inner wall of the annulus are used, the following expression is obtained:

$$\frac{\partial T^+}{\partial r^+} = \frac{\dot{q}''/\dot{q}''_{1,w}}{\dfrac{1}{Pr} - \dfrac{\varepsilon_M/\nu}{Pr_t}}. \tag{14}$$

The radial temperature distribution can therefore be calculated from the following equation:

$$T^+(r) = \frac{u_\tau}{\nu}Pr\int_{r_1}^{r}\frac{\dot{q}''/\dot{q}''_{1,w}}{1 - \dfrac{Pr\cdot\varepsilon_M}{Pr_t\cdot\nu}}\,dr. \tag{15}$$

3.5. *Stanton number calculation*

The bulk Stanton number is normally employed as the fundamental integral parameter for the heat-transfer calculation in simple duct channels. In case of unilateral heating in an annulus, i.e. under normal experimental situation the Stanton number is given by,

$$St_b = \frac{\dot{q}''_{1,w}}{(T_{w_1} - T_b)\rho\bar{u}c_p}. \tag{16}$$

The temperature difference $(T_{w_1} - T_b)$ can be obtained if the mean dimensionless temperature is calculated over the entire channel.

$$\bar{T}^+ = \frac{2}{\bar{u}(r_2^2 - r_1^2)}\int_{r_1}^{r_2} T^+ ur\,dr. \tag{17}$$

Considering the definitions of the bulk and the dimensionless temperatures, the following relation between $\bar{T}^+$ and $(T_{w_1} - T_b)$ is evident:

$$\bar{T}^+ = (T_{w_1} - T_b)\frac{\rho u_{\tau 1} c_p}{\dot{q}''_{1,w}}. \tag{18}$$

Replacing the mean temperature difference in equation (16) according to equation (18), the bulk Stanton number is a function of the mean dimensionless velocity and temperature values:

$$St_b = \frac{1}{\bar{T}^+\bar{u}}. \tag{19}$$

Under the hypothetical transformation conditions the outer wall is also heated. The corresponding two Stanton numbers are then given by:

$$St_{b_1} = \frac{1}{\bar{T}_1^+\bar{u}_1^+} \tag{20}$$

$$St_{b_2} = \frac{1}{\bar{T}_2^+\bar{u}_2^+}. \tag{21}$$

The two mean dimensionless velocities and temperatures in equations (20) and (21) are calculated between the corresponding wall and the radius of the adiabatic surface.

4. SOLUTION OF ANNULAR EQUATION

The quality of the solution obtained with the new method depends considerably on the accuracy of the temperature distribution calculated from equation (15). The choice of the eddy diffusivity of momentum and the turbulent Prandtl number which appear in this equation is therefore of great importance.

4.1. *Eddy diffusivity of momentum*

Many investigators proposed analytical expressions for the distribution of radial eddy viscosity in the simple channel geometries. One of the most known is that given by Reichardt [19] for smooth pipe flow. However, no satisfactory expression exists which would be completely consistent with the physical conditions of the turbulent flow in different channels. Appreciable deviations appear mostly in the central region. The most important constraints for the ε_M distribution giving an adequate expression for the turbulent velocity distribution can be generally summarised as follows:

1. The eddy diffusivity should vanish at the wall;
2. The analytical velocity derivative should give a zero value in the central region of the channel i.e. at the experimentally determined position of maximum velocity;
3. The continuity equation should be satisfied in such a way, that the integration of the predicted velocity distribution leads to the correct mean velocity;
4. The shear stress distribution should give acceptable friction factors with its Reynolds number dependency;
5. The calculated velocity distribution should agree with the best available experimental data, particularly in the central channel region.

All these conditions can be fulfilled for the tube and other simple channels with symmetrical velocity distribution. In case of an annulus, the conditions 2, 4 and 5 must be separately checked so that the corresponding ε_M distribution matches the experimental results.

The eddy viscosity distribution in the annulus can be determined experimentally if the pressure drop and velocity profiles are measured (condition 2).

$$\varepsilon_M = \frac{1}{2}\frac{dp}{dx}\frac{r_m^2 - r^2}{r\rho \dfrac{du}{dr}} - \nu. \tag{22}$$

The measurements of different investigators [20–23] show that the radial variations of the ε_M in an annulus is very similar to those of tube experiments. For moderate radius ratios of practical interest ($0.28 < d_1/d_2 < 0.75$) the same distribution of the normalized dimensionless eddy viscosity $(\varepsilon_M/\nu)/y_m^+$ was observed by Jonsson and Sparrow [20]. The curves have a similar form with the ones proposed by Reichardt for smooth tubes. It is normally assumed that the positions of zero shear stress and maximum velocity surfaces coincide, which is a good approximation for a smooth annulus with moderate radius ratio. In case of a partially rough annulus the displacement between these two positions is much larger. In order to fulfil the condition 4 the expression for the Reynolds stress must be corrected:

$$\overline{\rho u'v'} = \rho\varepsilon_M \frac{du}{dr} + \Delta\tau. \tag{23}$$

The shear stress distribution is given by

$$\tau = \mu \frac{du}{dr}\left(1 + \frac{\varepsilon_M}{\nu}\right) + \Delta\tau. \tag{24}$$

From the momentum balance, the approximate equation of the shear stress distribution in an annulus is determined by

$$\tau = \frac{1}{2}\frac{dp}{dr}\frac{r_0^2 - r^2}{r}. \tag{25}$$

Combination of equation (22), (24) and (25) leads to the expression for the correction term $\Delta\tau$:

$$\Delta\tau = \frac{1}{2}\frac{dp}{dx}\frac{r_0^2 - r_m^2}{r}. \tag{26}$$

The changes of ε_M with the Reynolds number can be determined by matching the measured velocity profiles (combination of the conditions 4 and 5), i.e. the ε_M values are to be changed until a satisfactory agreement between calculated and measured velocities is achieved.

4.2. *Turbulent Prandtl number*

Relatively few experimental investigations have been carried out to determine the temperature profile for heat transfer in turbulent flow. These profiles are needed, together with the distribution of eddy viscosity to calculate the turbulent Prandtl numbers. The first statement of Reynolds [24] using $Pr_t = 1$ has been questioned by many authors. Numerous attempts to obtain direct experimental evidence show no uniform and consistent results [25]. Most of the measurements are available for gases for which the molecular Prandtl number is close to unity. Ludwig [26] measured the variation of Pr_t with the radius of the tube for air. The measurements indicate that Pr_t varies smoothly and continuously from the value of about 0.9 at the wall, to about 0.6 at the center. These results are also in agreement with Taylor's vorticity theory [27]. Most of the existing theories assume $Pr_t = 1$ or an average and constant value. The results obtained by such assumptions are acceptably close to the measurements and suitable for most practical purposes. Gowen and Smith [28] investigated the change in Pr_t numbers against the radial position for turbulent air flow in smooth and rough tubes. The results of smooth tubes agree very well with those of Ludwig. Additionally important information about the effect of the roughness on the Pr_t was obtained. The changes in the radial direction are slightly more pronounced than in the smooth tube and the Pr_t values are generally (for about 20%) higher. The effect of the Reynolds number was investigated by different authors [29–31] and found to be important.

4.3. *Particular solutions*

To solve the annulus equations, two procedures were investigated:

Numerical integration; and
Simplified solution with direct, analytical integration.

Numerical integration. The main emphasis of this solution is to verify the quality of different other simplified methods for the evaluation of measured results. The development of the corresponding computer code NANOC was initiated after some encouraging results were obtained by Lawn with the

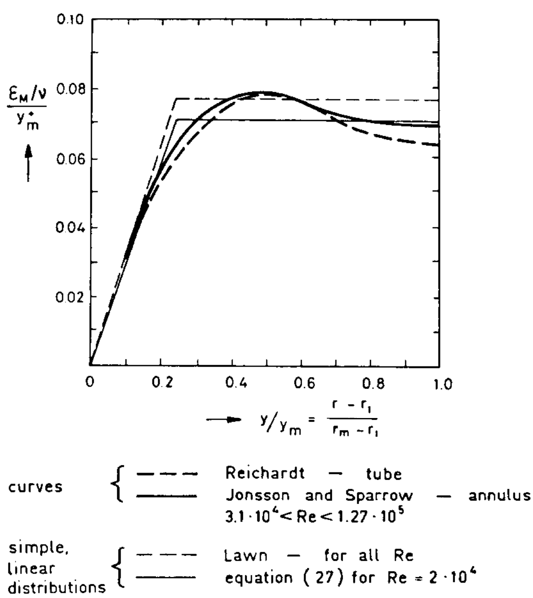


FIG. 3. Approximate values of eddy diffusivity used in different calculation models.

CONAN code [18]. Because of the subdivision of the annulus in very small calculation zones, the local conditions in the radial direction are considered. The velocity and temperature profiles are obtained by means of the "eddy diffusivity" concept for momentum and heat transport. The values of the eddy viscosity were determined as the best fit of the available experimental investigations [18–20, 23, 32, 33]. The typical curves according to different experimental informations are presented in Fig. 3. A curve similar to that of Jonsson and Sparrow is chosen for the numerical integration. Different turbulent Prandtl numbers for smooth and rough subchannels, according to the measurements of Gowen and Smith are taken.

Simplified solution. NANOC is a relatively large computer code requiring a lot of computer time. Therefore it is not suitable as a standard evaluation method for extensive measurement programs. The following simplifications are introduced, concerning both the calculation procedure and the radial distribution of the important quantities, to obtain a simplified computer code STANTRA:

The "eddy diffusivity" concept is applied only for the solution of the energy equation;

The numerical integration is replaced by a direct analytical integration. This was made possible by dividing the channel in a few zones where the local values of velocity, eddy diffusivity and turbulent Prandtl number can be represented by simple analytical functions;

The eddy diffusivity distribution, linear near the wall and constant in the core flow is presented in Fig. 3 for $Re = 2 \cdot 10^4$. For Reynolds numbers higher than 10^5 the distribution is approximately equal to those successfully used by Lawn [18]. The Reynolds dependency on the ε_M values in the core flow ($y/y_m > 0.23$) is determined by the following equation:

$$\frac{\varepsilon_M/\nu}{y_m^+} = 0.076\left(1 - \frac{1500}{Re}\right); \qquad (27)$$

The turbulent Prandtl number is assumed to be constant in each of the channel zones. These values are based on the radial distribution measured by Gowen and Smith [28].

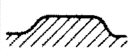
The same procedure as was developed for the evaluation of the annular experimental results is also applied for the calculations of temperature distribution in smooth and rough rod bundles. For these calculations a corresponding subroutine FASTCAL is developed and incorporated in the two subchannel analysis codes SCRIMP and CLUHET.

5. APPROVAL OF THE METHOD

5.1. *Comparison of the different transformation methods*

In order to compare the results of the different transformation methods some results of the experimental investigations with air [14] are revaluated. The results of the rod 26 with the roughness of rectangular rib profile, measured at the highest h^+ value are used for these calculations. The characteristics of the roughness shape are outlined in Table 1.

Table 1. Roughness characteristics

Rib dimensions					
h [mm]	$\frac{p-w}{h}$	$\frac{w}{h}$	Typical profile	Roughening method	Rod no. Ref.: [14] [36]
$1.58 \cdot 10^{-4}$	9.4	2.1		mecanical roughening	26
$1.0 \cdot 10^{-4}$	8.5	3.5		electro chemical grinding	215 251

The rectangular profile has been investigated to a large extent and was chosen because of the great amount of available experimental data. All transformation methods shortly described in Section 1.3, together with the new method, are applied. The friction data for the entire annulus are transformed for the inner subchannel according to the simplified Hall method. These transformed results were found to be very close to the results of other established methods for friction factor transformation and are therefore used as a starting point for all *St* transformation methods. Other important data are summarised as follows:

$$r_1/r_2 = 0.586 \qquad r_1/r_0 = 0.658$$
$$Pr = 0.702 \qquad T_w/T_b = 1.099$$
$$Re_1 = 5.32 \cdot 10^4 \qquad f_1 = 1.075 \cdot 10^{-1}$$
$$h^+ = 91.5 \qquad R = 4.64.$$

The roughness function *G* and the transformed Stanton number for the inner, rough subchannel of the annulus are calculated. In order to obtain the Stanton number multipliers, the transformed Stanton numbers for the smooth inner subchannel are also calculated. The method applied to the rough surface is also used for the smooth Stanton number transformation. This is done to ensure the consistency of the data. The transformed rough Stanton number is also related to the corresponding smooth tube value. This type of multiplier was often used by other investigators. For the transformation methods where the *G* function was not originally introduced, the value of this function is calculated from the heat transfer similarity law of Dipprey and Sabersky [8]

$$G = R + \frac{\frac{f_1}{8 \cdot St_1} - 1}{\sqrt{f_1/8}}. \quad (28)$$

In addition to the simplified solution some NANOC calculations are also carried out. This code is still under development and with the present version, only the R and G values can be evaluated.

The results of the different calculations are summarised in Table 2 together with the percentage of the deviation from the results of the new transformation method.

Table 2. Comparison of results of different transformation methods

METHODS / QUANT.		1 NEW METHOD	2 HALL - KJELLSTRÖM	3 DALLE DONNE - MEYER	4 RAPIER	5 NATHAN - PIRIE	6 NANOC VERSION of new method
G		11.27	10.87	12.78	12.17	12.30	11.10
	Δ %	0 (BASE)	- 3.6	+ 13.5	+ 8.1	+ 9.2	- 1.4
$St_1 \cdot 10^3$		7.06	7.80	6.80	7.18	7.12	
	Δ %	0	+ 10.5	- 3.9	+ 1.7	+ 0.8	
$St_{1s} \cdot 10^3$		3.57	3.72	3.43	4.00	3.54	
	Δ %	0	+ 4.2	- 4.1	+ 12.0	- 0.8	
Stx		1.98	2.1	1.98	1.79	2.01	
	Δ %	0	+ 5.9	0	- 10.6	+ 1.5	
Stx_t		2.14	2.36	2.06	2.17	2.16	
	Δ %	0	+ 10.3	- 4.0	+ 1.4	+ 0.9	

5.2. *Evaluation of the heat transfer roughness function for surface roughness with rounded trapezoidal ribs*

In order to choose a suitable rough surface for the application in gas cooled fast reactor (GCFR) core, different roughness shapes were tested with air in a simple single rod experiment. Based on a relative evaluation of the results, it was found that the rounded ribs of trapezoidal profile perform better than the sharp edged rectangular ones. The trapezoidal type of rib was therefore proposed for the use in the GCFR [34,35]. To obtain more accurate absolute values of the roughness functions, additional single rod tests with various gases under high pressure (~ 40 bar) and high temperature (up to 920 K) conditions, were carried out [36]. The roughness characteristics of the two rods tested are given in Table 1. The heat transfer results, obtained in CO_2 for the wall to bulk temperature ratios between 1.1 and 1.6, were transformed according to the new method. The evaluated roughness function G is presented in Fig. 4 and can be approximated for the range of h^+ between 25 and 300 by the following equation:

$$G = 4.5 \cdot h^{+0.24} \cdot Pr^{0.44}. \quad (29)$$

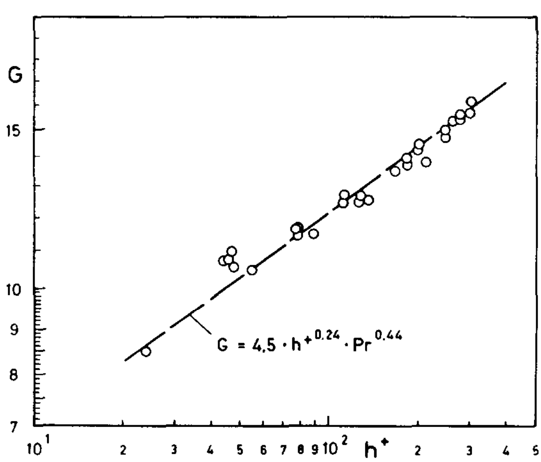


FIG. 4. Roughness function G for ribs with rounded trapezoidal profile, evaluated from the single rod experiment in CO_2.

5.3. *Application of the evaluated heat transfer roughness function to the rod bundle calculations*

The evaluated heat transfer function G was used in the subchannel analysis computer code SCRIMP [37] in order to predict the wall temperature distribution in a 37 rod bundle of hexagonal cross section. The predicted values can be compared with the results measured in the AGATHE HEX experiment [38]. In this experiment, with CO_2 as a coolant, a typical GCFR fuel assembly was simulated with 37 electrically heated rods. The measured local wall temperatures, under fully developed turbulent flow conditions, are presented for the two different Reynolds numbers in Figs. 5 and 6 together with their analytical predictions.

6. DISCUSSION OF THE RESULTS

Comparing the results of different transformation methods (Table 2), it can be seen that the G values obtained with the simplified version of the new method is slightly higher than calculated by the

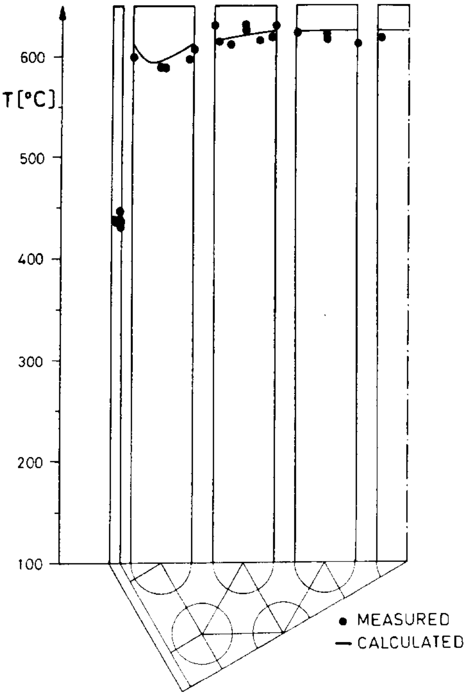


FIG. 5. Comparison between measured and calculated temperature distribution in a hexagonal 37 rod bundle at high Reynolds number $Re = 3.35 \cdot 10^5$: $T_{in} = 104°C$, $P = 39.6 \cdot 10^5\,N\,m^{-2}$, $\dot{q}'' = 75.8\,W\,cm^{-2}$.

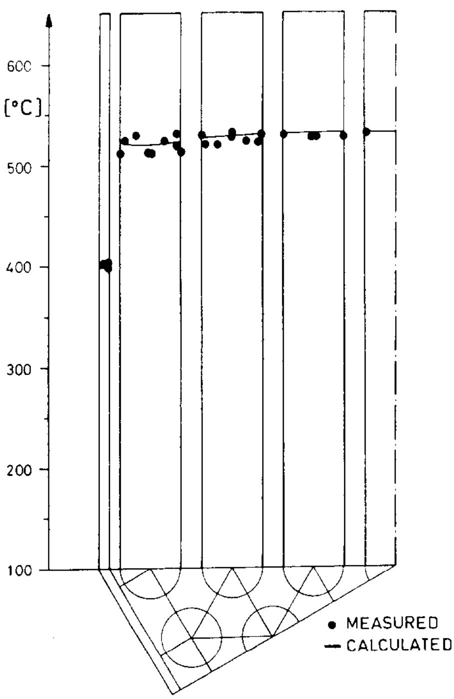


FIG. 6. Comparison between measured and calculated temperature distribution in a hexagonal 37 rod bundle at low Reynolds number $Re = 3.5 \cdot 10^4$: $T_{in} = 103°C$, $P = 6.7 \cdot 10^5\,N\,m^{-2}$, $\dot{q}'' = 6.5\,W\,cm^{-2}$.

Hall–Kjellström method, but very close to the results of the NANOC code. The G values of other methods are considerably higher. The highest deviation of $+13.5\%$ is noticed for the method of Dalle Donne–Meyer.

A very good agreement in the absolute values of the transformed rough Stanton numbers is reached between the method developed by the author and the methods of Rapier and Nathan–Pirie. The simplified Hall–Kjellström method gives considerably higher values and the method of Dalle Donne–Meyer slightly lower values. Similar behaviour can be established for the transformed smooth values except for Rapier's method which seems to be unfit for the transformation of the smooth Stanton numbers. For other than Rapier's method an acceptable scatter of the annular multiliers is observed. For the multipliers related to the tube St values, the same conclusion is evident as for the absolute values of the transformed rough Stanton numbers.

Based on these comparisons, the simplified Hall–Kjellström method and the method of Rapier cannot be generally recommended for further use, but may be suitable for simple calculations in particular cases.

For the application of the heat transfer data by the hydraulic diameter concept the method of Nathan and Pirie seems to be very useful. This calculation is very simple. However, because of the empirical basis of the method, the validity of the transformation for other than rectangular ribs should be additionally confirmed.

The method of Dalle Donne and Meyer gives generally the lowest heat-transfer performances (highest G and lowest St_1 values). However, if this method is consistently used, the deviation in the G function is not so important, at least for the high Reynolds number. The use of the G values calculated from the general correlation for rectangular ribs given by Dalle Donne and Meyer [11] is restricted to their method. Applied to other methods these values cause an additional decrease of heat transfer performances which may produce too pessimistic results.

The analytically predicted local temperature distribution in a hexagonal 37 rod bundle, based on the application of the G function estimated by the single rod experiment, are in good agreement with the experimental information. The deviation between the measured and predicted wall temperatures the bundles are less than $\pm 3\%$ of the total temperature increase $(T_w - T_{in})$.

Before the decision about a unique basic transformation method is made, further investigations are needed. These should be carried out under the particular conditions of lower Reynolds numbers, different ratios of the heat fluxes at the walls, different channel forms etc. The verification of these methods with the extensive experimental investigation in the bundle channel geometry may therefore be of a decisive importance.

7. CONCLUSIONS

Based on the solution of the simplified differential equation of the core flow, a new method for the transformation of turbulent Stanton numbers measured in the annular channel geometry was developed.

To test this method some results of a single rod experiment are revaluated. Based on the comparison of these results with those of different other transformation methods, the following conclusions can be drawn:

No final decision about the best transformation method can be made at the present time. Together with the new method described here, the transformations of Dalle Donne–Meyer and Nathan–Pirie can be recommended for the final selection;

The values of the G function calculated by these three methods differ considerably but provided these methods are consistently used, the absolute values of the transformed Stanton numbers are within an acceptable range;

The new method has the highest generalisation and development potentials. Not enough experience with this method is available to date, but further investigations will probably show whether the expected advantages can be confirmed. Additional experimental information concerning the eddy diffusivity of momentum and the turbulent Prandtl number may still increase the confidence in the method and improve its accuracy under complicated flow and heat transfer conditions;

The heat-transfer function G, evaluated by the new method, was applied to predict the temperature distribution in a hexagonal 37 rod bundle. The analytical results agree very well with the measured temperatures, the deviations being always less than $\pm 3\%$.

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EVALUATION DES PERFORMANCES DE TRANSFERT DE CHALEUR DES SURFACES RUGUEUSES A PARTIR D'ETUDES EXPERIMENTALES DANS DES CANAUX ANNULAIRES

Résumé—Une nouvelle méthode a été développée, pour la transformation des nombres de Stanton turbulents, mesurés dans des canaux à géométrie annulaire. La méthode est basée sur la solution de l'équation différentielle simplifiée du transport d'Energie en écoulement turbulent. Pour contrôler cette méthode, différents calculs de transfert de chaleur sont effectués dans des canaux à surfaces lisses et rugueuses. A titre de comparaison, d'autres procédures de transformation sont aussi utilisées. Les performances thermiques d'une rugosité particulière, évaluées à partir de l'étude expérimentale d'un barreau isolé, sont utilisées dans un programme d'ordinateur afin de calculer la distribution des températures dans un faisceau de 37 barreaux. Les prédictions sont comparées aux résultats expérimentaux.

BESTIMMUNG DER WÄRMEÜBERGANGSEIGENSCHAFTEN RAUHER OBERFLÄCHEN AUS DER EXPERIMENTELLEN UNTERSUCHUNG IN RINGKANÄLEN

Zusammenfassung—Eine neue Methode für die Transformation der im Ringkanal gemessenen turbulenten Stanton-Zahlen wurde entwickelt. Die Methode basiert auf der Lösung der vereinfachten Differentialgleichungen des turbulenten Wärmeaustausches in der Strömung. Um die Methode zu prüfen, wurden verschiedene Wärmeübergangsrechnungen in Kanälen mit glatten und rauhen Oberflächen durchgeführt. Zum Vergleich wurden einige andere Transformationsmethoden angewandt. Die vom Einzelstabexperiment gewonnenen thermischen Eigenschaften einer bestimmten Rauhigkeit wurden in einem Rechenprogramm verwendet, um die Temperaturverteilung in einem 37-Stab-Bündel zu berechnen. Die analytischen Voraussagen wurden mit den experimentellen Resultaten verglichen.

ОЦЕНКА ХАРАКТЕРИСТИК ТЕПЛООБМЕНА ШЕРОХОВАТЫХ ПОВЕРХНОСТЕЙ ПО РЕЗУЛЬТАТАМ ЭКСПЕРИМЕНТАЛЬНОГО ИССЛЕДОВАНИЯ КОЛЬЦЕВЫХ КАНАЛОВ

Аннотация — Разработан новый метод определения турбулентных чисел Стэнтона, измеренных в кольцевых каналах. Метод основан на решении упрощенного дифференциального уравнения переноса энергии турбулентного потока. Для проверки метода выполнены расчёты процессов теплообмена в каналах с гладкими и шероховатыми стенками. Для сравнения использованы также другие методы преобразования. Тепловые характеристики шероховатости конкретного типа, определенные из эксперимента на единичном стержне, заложены в программу ЭВМ для расчёта распределений температур в пучке из 37 стержней. Проведено сравнение результатов аналитических расчётов с экспериментальными данными.